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原著

A Study on the Development of Regional Master Recession Curve Model

Jae-Hyoung Lee[1], Nam-Sun Oh[2], and Hee-Ju Lee[3]

[1]Department of Civil Engineering, Chonbuk National University, ChonJu, Korea
[2]Department of Ocean Civil Engineering, Mokpo National Maritime University, Mokpo, Korea
[3]Department of Civil Engineering, Chonbuk National University, ChonJu, Korea

ABSTRACT

A regional master recession curve model to predict groundwater discharges in a given basin was presented. Considering a stream-aquifer system, both theoretical and experimental baseflow equations were compared and a practical groundwater discharge equation was derived. The groundwater discharge equation was expanded and transformed to the discharge equation at the basin exit. For practical use, the equation was expressed as a function of watershed area, the mean slope of basin and the recession constant. To verify the model, the model was applied to Ssang-chi basin where long-term and temporal hydrological data at the upper basin were collected. Our results show that a master recession curve of unmeasured area can be predicted.

Key words : regional master recession curve model, groundwater discharge equation, stream-aquifer system

1. INTRODUCTION

The estimation of groundwater discharge from the aquifer to the stream is very important in terms of water quality management, water supply planning, irrigation planning and determination of stream maintenance flow[1]. Barns[2], Boussinesq[3] and Maillet[4] began studies on the evaluation of groundwater store discharges in the late 19th century and Hall[5], Appleby[6], Brutsaert and Nieber[7], Vogel and Kroll[8], Troch[9] et al. have studied. Usually such studies focused on low flow evaluation, characteristics of groundwater runoff in the aquifer and evaluation of groundwater storage volume. However, a more appropriate model was not yet developed for the practical use. Specially, groundwater discharge depends on the uncertainty of groundwater flow and the topographic and geologic characteristics of basin and groundwater storage volume and basin storage volume[10]. Several groundwater runoff equations were developed and expressed as a function of a permeability coefficient, porosity and initial discharge, but such variables can not be easily obtained. So, suggested equations have not yet been used for practical purpose. The overall objective of this study is to develop a regional master recession curve model which can be used easily in practice.

2. BASEFLOW

The baseflow in a natural river system can be defined as the flow resulting from the drainage from the groundwater aquifer in the basin. The base flow volume from a given basin depends not only on basin characteristics but also aquifer characteristics, groundwater storage and stream stage. The baseflow mainly depends not on meteorological characteristics but geologic and topographic characteristics. The model, which involves such concepts on baseflow, will be derived in this chapter.

2.1 Baseflow equation

Accurate estimation of groundwater discharge is difficult because the relationship between groundwater flow and geologic, topographic characteristics is complicated. Many studies on such relationship have been conducted showing that initial discharge and recession parameter can be replaced by geologic and topographic parameters[11].

Dupuit introduced the hydraulic approach to the calculation of groundwater discharge in the early 1860's, while Boussinesq suggested the basic nonlinear differential equation governing unsteady flow from a large unconfined aquifer to a stream channel. Based on Dupuit's assumption which neglects vertical flow, Boussinesq derived a linear form of the flow equation. This equation is valid under the idealized conditions of no evapotranspiration, leakage or recharge. Many theoretical equations for determining groundwater flow based on the Boussinesq equation have been presented by Singh[12, 13], Brutsaert and Nieber, and Zecharias and Brutsaert[14] while some equations were derived from Richard equation. Groundwater flow equations were presented in a linear or nonlinear form and Brutsaert and Nieber suggested discharge equation in both linear and nonlinear form. A linear equation can be expressed by the function of initial discharge and recession constant. A linear equation is simpler than a nonlinear equation and often used for the evaluation of baseflow discharge. A linear equation by Brutsaert and Nieber is similar to the baseflow equation presented by Barns and has recession constant of exponential form. Equation (1) is a linear discharge equation, which is expressed by initial discharge Q_0 and recession constant .

$$Q = Q_0 K^t \qquad (1)$$

Lee[15] computed and compared a baseflow discharge from linear discharge equation (1) and Brutsaert and Nieber's nonlinear equation at Ssang-Chi basin. His results indicate that application of both equations shows little difference from an engineering point of view. This means that linear equation is simple and very promising for practical use.

2.2 Stream-aquifer model

The initial discharge in equation (1) can be evaluated when long-term measured data can be obtained. But long-term measured data can not be easily obtained. Using the stream-aquifer system approach, the initial discharge in a basin can be replaced by watershed characteristic constants which are easily obtained.

Analogous to Brutsaert and Nieber's method, the stream-aquifer system(Fig. 1) is conceptualized as outflow into a fully penetrating stream channel from an unconfined aquifer overlying on a impermeable layer. Stream-aquifer system is classified as both a horizontal and an inclined system.

Considering evapotranspiration, Zecharias and Brutsaert〔ZB eq.〕 presented a discharge equation with an assumption that groundwater flow is steady for an inclined aquifer. Using the linearized Boussinesq equation, Wilfried Brutsaert[16]〔WB eq.〕 suggested a groundwater discharge equation. Using Fourier method, Boussinesq presented linear and nonlinear equation for a horizontal aquifer. Also, Poubarinova-Kochina[17], Lembke suggested a groundwater discharge equation using the soil characteristic function.

The equation, which is applicable to both a horizontal and an inclined aquifer, was not yet suggested. To obtain a model which is applicable to both aquifers, equations suggested previously were compared to experimental data by Stanford[18]〔WS eq.〕. For a inclined aquifer, experimental data 〔WS eq.〕 were compared with the equation by Zecharias and Brutsaert and the equation by Brutsaert in Fig. 2(b) and Fig. 2(c). In the figures, values obtained from both equations coincide with experimental data well. ZB equation must evaluate evapotranspiration and is not practical due to its complexity. In a horizontal aquifer, Fig. 2(a) shows that a linearized

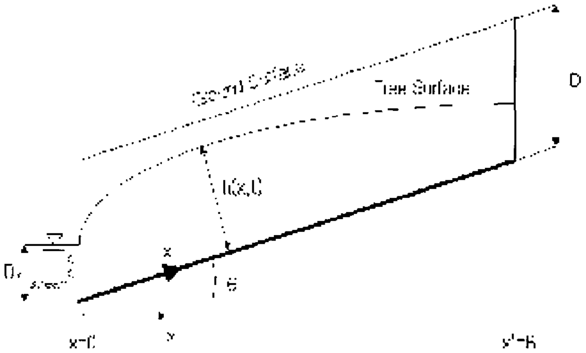


Fig. 1. Conceptualization of stream-aquifer model

Boussinesq equation yields better results than other equations.

The WB equation for an inclined aquifer is similar to the Boussinesq equation in a horizontal aquifer. When the slope of a aquifer is equal to zero, the WB equation has the same form as the linearized Boussinesq equation. This means that the WB equation can be commonly used, but the complexity of the equation makes its use difficult. So, we modified this simple Boussinesq equation instead of WB equation for the inclined case and fitted it to the experimental data18) in Fig 2.

$$q = 2kpD\left\{\frac{(D-D)}{B}\right\}\mathrm{esp}\left\{\frac{-\pi^2 kpDt}{4fB^2}(1-\tan\theta)\right\} \quad (2)$$

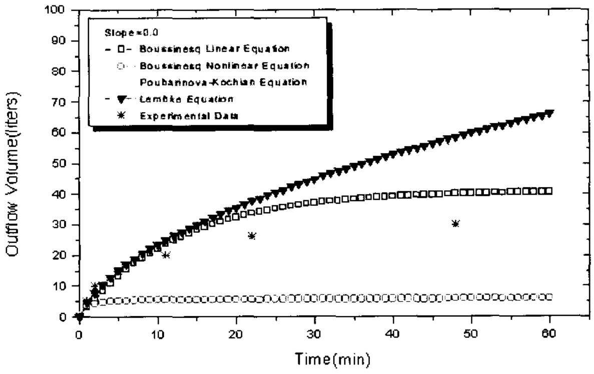
The Fig 2. shows that the equation (2) is well adapted to the data.

2.3 The baseflow model

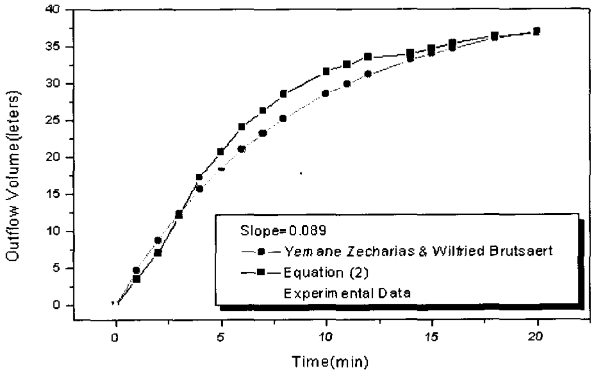
A watershed can be conceptualized as composing of a large number of stream-aquifer elements. The equation (2) may be extended to an entire watershed by employing a few hydrologic similarity conditions. For example, the total watershed runoff Q arises from the lateral inflow contributions to all stream channels; hence

$$Q = 2Lq \quad (3)$$

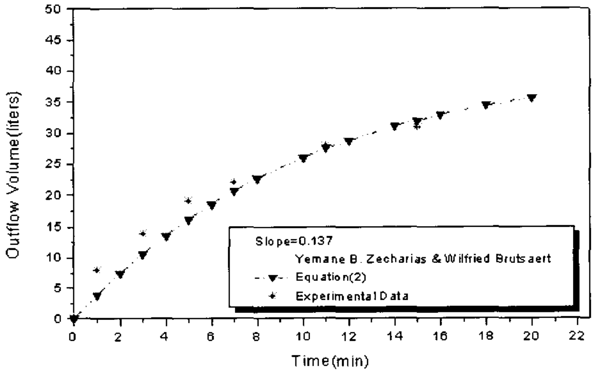
where L is the total length of all stream channels in the watershed. Fig. 3 shows the relationship between the lateral inflow from aquifers and the total watershed runoff. Parameters such as the hydraulic conductivity, the porosity of soil and the initial discharge in



(a) Horizontal aquifer



(b) Inclined aquifer(slope = 0.089)



(c) Inclined aquifer(slope = 0.137)

Fig. 2. The comparison of discharge equations and experimental data

equation (3) are geologic properties and can be easily measured.

Vogel and Kroll(1992) simplified the equation. The first and the second terms, initial discharge, in equation (3) were simplified by using watershed area and average slope of the land surface and the third term by using a recession constant. Introducing the concept suggested by Vogel and Kroll, equation (1), (2) and (3) can be combined to yield

$$Q = b_0 A^{b1} S^{b2} K_b^{b_3(t)} \epsilon^{b_4} \qquad (4)$$

where A is the watershed area, S is the average slope of the actual land surface and K_b is the baseflow recession constant.

Equation (4) introduced in this study can evaluate groundwater outflow at any point in the watershed using the watershed area, the average slope of watershed and the recession constant.

3. BASEFLOW CHARACTERISTICS

In equation (1), the initial discharge and the recession constant represent the baseflow characteristics of a watershed. In equation (4), baseflow is affected by watershed characteristics and runoff characteristics. In the equation, the watershed area and the average slope of watershed are watershed characteristics, and the initial discharge and the recession constant are runoff characteristics.

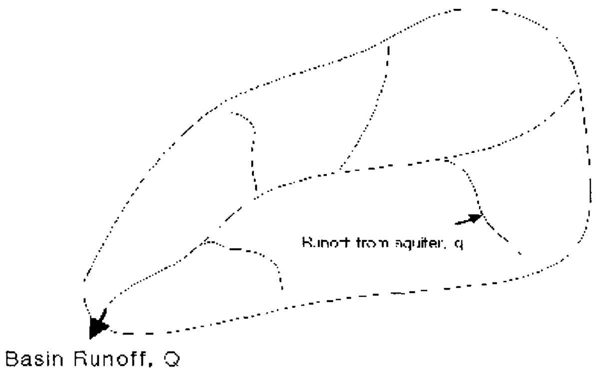


Fig. 3. The stream-aquifer system in the watershed

In this section, to estimate certain parameters in equation (4), watershed and runoff characteristics in a given area will be described.

3.1 Characteristics of watershed

The exit of the study area is located at Ssang-chi station, Chu-ryeong stream, Seomjin river basin. Gauging at Ssang-chi station started in December 1992. To establish a regional master recession curve model, measured flow data at the several points are needed. But, in the Ssang-chi basin, long-term data at the exit of basin and temporal data at the several points had already been collected. The measured 9 points are located in Hak-seon stream, Bang-san stream, Sang-song li, Bong-seo stream, Geum-weol stream, Gal-weol stream, Confluence of Hak-seon stream and Bang-san stream.

In general, watershed characteristics can be defined by the area of watershed, the shape of watershed and topography and geology. In this study, watershed characteristics used were the area of watershed, the total length of channel and the average slope of watershed (Table. 1). The area of the watershed in this study is 0.587-126.3㎢ and the average area is 17.569㎢. The average area of the upstream minor

Table. 1. The characteristics of watershed at the gauging station

minor watershed	Watershed area (㎢)	length of channel (㎞)	slope of area
Exit of main stream	126.30	36.0	0.809
Hak-seon stream	10.424	2.991	0.842
Bang-san stream	10.672	3.672	0.997
Sang-song li	2.108	2.389	0.589
Bong-seo stream	3.768	4.536	0.898
Dae-ga stream	9.426	11.081	0.866
Geum-weol stream	5.788	4.121	0.315
Gal-weol stream	7.212	4.909	0.966
Confluence of Hak-seon stream	0.667	0.759	0.832
Confluence of Bang-san stream	0.587	0.433	0.328
Average	17.659	7.089	0.744

watershed, excepting the main watershed, is 5.628㎢. Total length of channel is 0.433-36.0 ㎞ and the average length of channel is 7.089 ㎞. Average slope of watershed is 0.315-0.997 and its mean is 0.744. Most parts of watershed are composed of mountain and have steep slopes (Lee, 2000). Fig. 4 illustrates the gauging stations used in the studied watershed.

3.2 The runoff characteristics of main watershed

The recession of stream flow can be defined as the depleting process of groundwater after the recharging of stored surface flow is finished. So, the discharge of groundwater can be represented by the recession curve of the groundwater. Characteristics of recession curve can be determined by the recession constant. The recession constant varies with the runoff, watershed and geologic characteristics, and is always less than one. On the baseflow, Barnes defined that the recession constant varies from 0.93 to 0.995, Martin[19] from 0.80 to 0.99, Singh[20] from 0.9 to 1 and Tallaksen more than 0.7. The average recession period of the study area is 15.86 days and the master recession constant is 0.821, which can be considered to baseflow recession characteristics.

The initial discharge, the flow rate at the

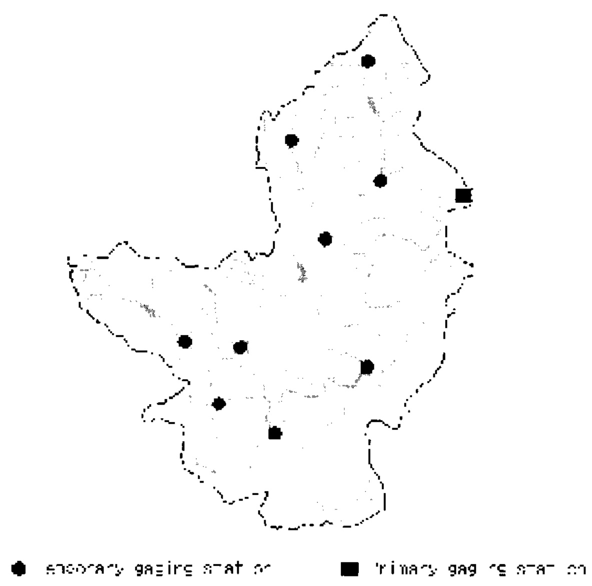


Fig. 4. Study area

time at which the groundwater recession curve begins, is the outflow at the state at which the aquifer is fully saturated. In this study area, the range of initial discharge is 0.78 129.44m³/sec. Because of the large variability of the initial discharge, it is difficult to estimate the initial discharge of recession curve. To obtain initial discharge, Troch used a graphical method which plotted the differential of flow rate to the flow rate and the time. Using Troch's method, calculated value of initial discharge is 3.339m³/sec and the time from initial discharge to the reference and the average drought flow in the equation (1) is 18.4 and 14.6 days, respectively.

3.3 The runoff characteristics of minor watershed

The long-term flow data are necessary to determine the model constant in the baseflow recession model equation (4). But, continuous measured data was not collected in the minor watershed station in this study area. The initial discharge and recession constant can not be estimated, due to a lack of data. But, the relationship between upstream station and downstream station can be obtained. Using this relationship, the characteristics of groundwater discharge can be calculated in the discontinuously measured station. This relationship is called the upstream-downstream relationship and has the form of the power function[21) . In this study area, the power index in the relationship, a and b, have the range 0.001-0.208 and 0.825-1.521, respectively. The initial discharge in the minor *watershed varies from* 0.005 *to* 0.641 m³/sec with an average of 0.120 m³/sec. The recession constant varies from 0.741 to 0.850 and its average is 0.797. The obtained recession constant satisfies the range defined in section 3.2. Table. 2 illustrates the runoff characteristics and power index, a and d, in the each minor watershed.

4. MODEL CONSTANTS

In this section, model constants in equation (4) are estimated. To accomplish this, the master recession curve at each station was calculated from the measured data. Minimizing the difference between master recession curve data and calculated value, model constants are estimated.

4.1 Regional master recession rate curve

Groundwater runoff can be predicted from equation (4). The model suggested in this study can be estimated with a sample master recession curve. By means of equation (1), the recession curve at the minor watershed exit can be synthesized with the initial discharge and the recession constant. In general, the initial recession time can be determined, but the last recession must be assumed. The recession is ended by means of another rainfall. After the recession continues for quite some time, the recession discharge is very small and shows little change. Considering that the design of low flow in this study area is 15~18 days, the period of data is determined to 30 days. Fig. 5 illustrates the synthesized recession curve at the exit of minor watershed.

Table. 2. The runoff characteristics in the each minor watershed.

minor watershed	initial discharge (cms)	recession constant	A	d
exit of main stream	3.393	0.821	-	-
Hak-seon stream	0.641	0.834	0.208	0.922
Bang-san stream	0.029	0.773	0.006	1.303
Sang-song li	0.027	0.806	0.007	1.094
Bong-seo stream	0.104	0.850	0.038	0.825
Dae-ga stream	0.018	0.783	0.004	1.241
Geum-weol stream	0.248	0.843	0.086	0.867
Gal-weol stream	0.006	0.741	0.001	1.521
Confluence of Hak-seon stream	0.005	0.769	0.001	1.333
Confluence of Bang-san stream	0.005	0.773	0.001	1.305
Average	0.120	0.797	0.04	1.16

4.2 The estimation of model constants

Characteristics of watershed and master recession curve at each station were determined up to here. The recession discharge at each station can be predicted by determining model constants in equation (4). These values must be coincided with the calculated data in Fig. 5. In order to estimate model constants, equation (4) is transformed to equation (5).

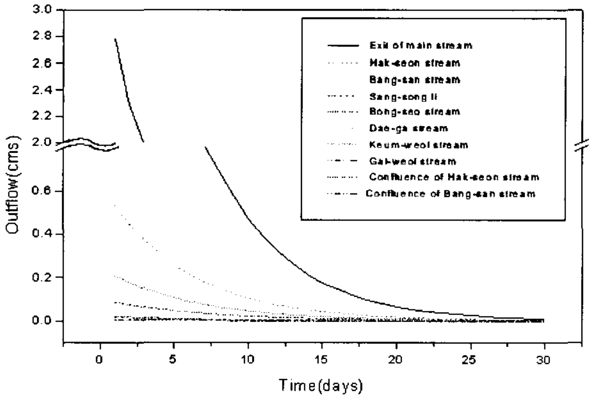


Fig. 5. Master recession curve at each station

$$\log Q = \log\alpha + \beta\log A\ \gamma\log S + \omega(t)\log K + \varepsilon \qquad (5)$$

where, $\alpha = \log b_0$, $\beta = b_1$, $\gamma = b_2$ and . $\omega(t) = b_3(t)$. A regression analysis was executed to estimate model constants α, β, γ, $\omega(t)$. Therefore α, β, γ was estimated to 0.66, 0.92 and -0.45, respectively and $\omega(t)$ was estimated to 27.5+t. The standard deviation was 0.307 and the coefficient of determination was 0.94. The coefficient transformed to original equation gave us b_0 = 4.57, b_1 =0.92, b_2 =-0.45 and $b_3(t)$ =27.5+t.

4.3 Discussion

In section 4.2, the estimation of model constants in the equation (4) resulted in the completion of the regional master recession curve model. In order to examine the cause of errors, the calculated value was compared with the recession curve data at each watershed in Fig. 6. In the figure, considerable error occurred at the initial discharge.

The watersheds where considerable errors occurred were main stream, Hak-seon stream, Bong-seo stream, Dae-ga stream, Geum-weol stream and Gal-weol stream. Stations which have large error have 2~24 flood plain, but stations which shows little error have no flood plain. Excepting Hak-seon stream, the station which have many flood plains have an inclination to have large error at the initial discharge.

The amount of initial discharge error is largest at the main stream and is smallest at the Sang-song-li, Hak-seon and Bang-san streams which have no flood plain. In Fig. 6, little error occurred after 14 days, the recession length of design low flow, from the initial recession. Consequently, the initial discharge is influenced by groundwater discharge. Groundwater discharge has little influence at levels below design low flow.

5. CONCLUSION

Using characteristics of watershed, a regional master groundwater recession curve model was suggested in this study. The suggested model is expressed as a function of watershed area, average slope of watershed and the recession constant. The model is convenient to use and can be used at an unmeasured watershed. The model was developed based on the baseflow equation and stream-aquifer relationship. Model constants were estimated by using runoff characteristics at the upstream minor watershed of Ssang-chi basin. Using groundwater characteristics of main stream and the upstream-downstream relationship, the initial discharge and the recession constant were estimated.

The area of main watershed was 126.3㎢. The area of minor watershed was 0.587-10.672 ㎢ with an average of 5.628㎢. The mean slope of watershed was 0.315-0.997 with an average of 0.744. The initial discharge in the exit of main watershed was 3.393㎥/sec, in the minor watershed the initial discharge was 0.005-0.641 ㎥/sec with an average of 0.120. The recession constant was 0.741-0.850 with an average of 0.799.

Using characteristics of watershed, model constants were estimated and the decision coefficient was 0.94. The calculated and observed values were compared and show considerable error at the initial recession stage. It is considered that errors at the initial stage are due to flood plain of upstream. Little error occurred at the period after 14 days from the initial recession, the recession length of design low flow. Consequently, the regional master recession curve model suggested here would be useful in the prediction of low flow at the unmeasured watershed or geologically similar area.

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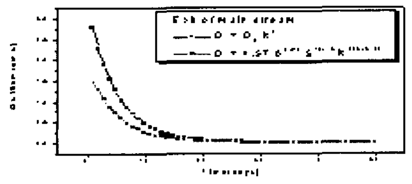

Fig. 6(a). exit of main stream

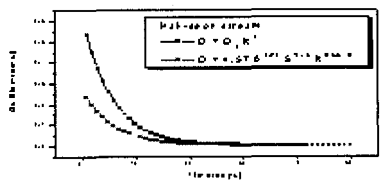

Fig. 6(b). exit of Hak-seon stream

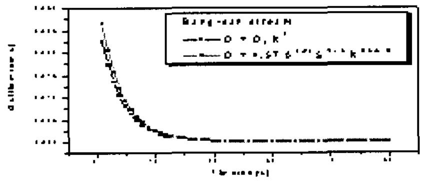

Fig. 6(c). exit of Bang-san stream

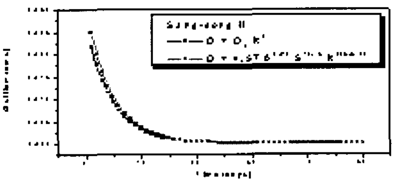

Fig. 6(d). exit of sang-song li

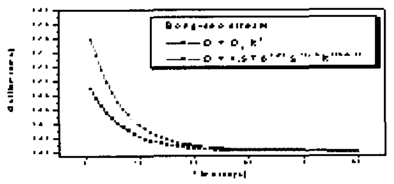

Fig. 6(e). exit of Bong-seo stream

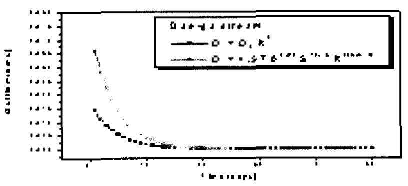

Fig. 6(f). exit of Dae-ga sream

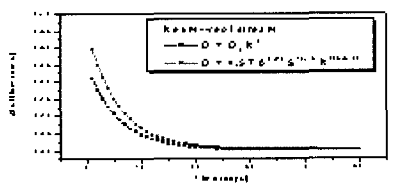

Fig. 6(g). exit of Geum-weol stream

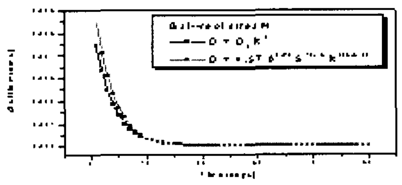

Fig. 6(h). exit of Gal-weol stream

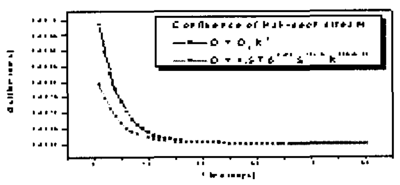

Fig. 6(i). confluence of Hak-seon stream

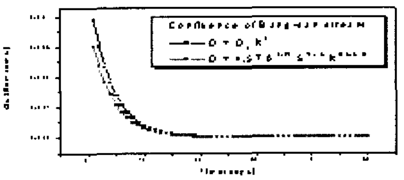

Fig. 6(j). confluence of Bang-san stream

Fig. 6. The comparison of recession curve

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